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Low-Energy-Consumption Hybrid Vibration Suppression Based on an Energy-Recycling Approach

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An innovative method of hybrid vibration suppression using piezoelectric materials is proposed. It combines bang–bang active vibration suppression and energy-recycling semiactive vibration suppression. The piezoelectric materials are electromechanically coupled and convert mechanical energy into electrical energy and vice versa. With this method, a part of the electrical energy needed for suppressing vibration is obtained from the mechanical energy of the vibrating structures and is efficiently recycled. Furthermore, the actively supplied energy is stored in the transducers and is reused many times for vibration suppression. Therefore, the hybrid method has better performance than the case where the bang–bang active method and the energy-recycling semiactive method are both used, but independently. The hybrid method saves the actively supplied energy and is thus a low-energy-consumption vibration control. Its effectiveness in suppressing vibrations was proven in numerical simulations and experiments using a 10-bay truss structure. Moreover, a novel method to prevent undesired control chattering is proposed to further save energy supplied from the external source.

Nomenclature

$\boldsymbol{B}_p$ = input matrix
b_p = piezoelectric constant of piezoelectric transducer
$\boldsymbol{C}_p$ = diagonal constant-elongation capacitance matrix
C_p^S = constant-elongation capacitance of piezoelectric transducer
I_{rms} = performance index in simulations; Eq. (28)
$I_{2\mathrm{rms}}$ = performance index in experiments; Eq. (42)
$\boldsymbol{K}$ = constant-charge stiffness matrix of structure
k_p = constant-charge stiffness of piezoelectric transducer
L = inductance in electric circuit
$\boldsymbol{L}$ = diagonal inductance matrix
$\boldsymbol{M}$ = mass matrix of structure
Q = electric charge given to piezoelectric transducer
$\boldsymbol{Q}$ = charge vector
$\boldsymbol{Q}_T$ = target charge vector obtained from active control
$\boldsymbol{q}$ = modal displacement vector
R = electric resistance in electric circuit
$\boldsymbol{R}$ = diagonal resistance matrix
u_1, u_2 = x-directional displacements at tip and central nodes
V_a = voltage generated by piezoelectric effect
V_{ext} = externally supplied voltage
V_p = voltage across piezoelectric transducer
$\boldsymbol{V}_p$ = voltage vector
V_{ref} = reference voltage for chattering prevention
$\boldsymbol{V}_1, \boldsymbol{V}_2$ = noise intensity matrices for Eqs. (33) and (35), respectively
$\boldsymbol{W}_1, \boldsymbol{W}_2$ = weighting matrices; Eq. (12)
$\boldsymbol{w}$ = external force vector
$\boldsymbol{x}$ = displacement vector of structure
$\boldsymbol{z}$ = state vector; Eq. (10)
δ_{rms} = rms of displacements of all truss nodes
ζ = modal damping coefficient
ϕ_i = eigenvector of ith vibration mode
ω_i = angular frequency of ith vibration mode

Subscripts and Superscript

j = jth piezoelectric transducer or electric circuit (for C_p^S, L, Q, Q_T, R, V_{ext}, V_p, and V_{ref})
p = piezoelectric transducer
$\hat{}$ = estimated value based on Kalman filter

I. Introduction

THERE are a large number of researches on synthesis of piezoelectric materials and electric devices to suppress structural vibrations.[1,2] Piezoelectric materials attached to or embedded in structures can convert mechanical energy into electrical energy or, conversely, electrical energy into mechanical energy. They have been extensively used as actuators, sensors, and transducers for various purposes.

For active vibration suppression, a voltage or an electric charge has to be supplied from an external energy source to piezoelectric actuators. However, for special structures that have limited energy sources, such as space structures and sea-based platforms, it is imperative to minimize energy consumption. For such a purpose, a number of hybrid vibration controls have been proposed. They are mostly combinations of active and passive methods. As for hybrid control with piezoelectric materials, several types of active-passive hybrid piezoelectric network (APPN) have been proposed.[3–6] This type of network integrates piezoelectric materials with an active voltage source and a passive shunting circuit composed of resistors and inductors. All are based on the passive vibration suppression proposed by Hagood and von Flotow[7] and Wu.[8] Here, an electric circuit connected with an inductor and a resistor has an electrical resonance that works to suppress the vibration in a similar way to dynamic mass dampers. However, to tune the electrical frequency to the quite low frequencies of structural vibrations, a large and heavy inductor is required if an active emulating circuit is not used. The inductor is often too heavy, especially for low-frequency structures. More important, the performance of the passive method using inductive resonance can deteriorate because of system model errors; the electrical resonance can work only when the structural and electrical frequencies coincide exactly. This is why the APPN methods have limited robustness. Therefore, a new vibration suppression method has been sought that consumes little external energy as well as being robust against model errors. These characteristics are important for

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structures whose energy sources are limited and whose models are hard to identify accurately, such as large space structures.

Recently, some works on semiactive vibration suppression with a circuit switch have been proposed.[9–15] In their studies, the circuit switch is turned on and off to suppress the vibration of a structure. Clark[9] has investigated a switch-shunting method implemented with a switchable stiffness element. To avoid potentially undesirable mechanical transients, a state-switched vibration absorber was proposed by Holdhusen and Cunefare[10] and Larson and Cunefare.[11] In particular, Onoda et al.,[12] Corr and Clark,[13] and Richard et al.[14] proposed and investigated another semiactive vibration suppression method using an inductive circuit. A piezoelectric transducer is shunted to the circuit for a short time at each peak of vibration strain. It has a significant feature; the electric energy stored in the transducer is recycled rather than immediately being dissipated.[15]

The first objective of this work is to develop a hybrid vibration suppression method that combines the bang–bang active method and the energy-recycling semiactive method proposed by Onoda et al.[12] The bang–bang active method has a control input of one absolute value and changes only the polarity of the control input, unlike continuous feedback methods. The voltage supplier can be shared by multiple actuators, which can simplify the hardware configuration. Although the energy-recycling semiactive method has been shown to be much better at vibration suppression than a non-energy-recycling semiactive method,[12–15] it is limited because it only uses converted electrical energy. The hybrid method can enhance the vibration suppression performance of the bang–bang active method because it can convert some of the mechanical energy into electrical energy for vibration control, instead of this electrical energy being provided entirely from external sources. Moreover, the externally provided energy is also stored in the transducer and is reused to suppress structural vibrations. Thus, the proposed hybrid method not only recycles the converted electrical energy, but also prevents the supplied external energy from being dissipating so that it can be used later on. Therefore, the hybrid method might have better performance than the case where the bang–bang active method and the energy-recycling semiactive method are both used, but independently. Because it reduces the actively supplied energy, it can be categorized as a low-energy-consumption vibration control. Furthermore, because it performs in synchronization with the phase of structural vibration, it should be more robust against model errors than the APPN methods using inductive resonance. Numerical simulations were carried out to evaluate its robustness against model errors of systems, as well as its vibration suppression capability. To investigate the actual behavior of MDOF systems with the hybrid method, vibration suppression experiments by using a 10-bay truss structure were performed.

In the energy-recycling semiactive vibration suppression, after the vibration has been sufficiently suppressed the control input value can become much larger than necessary. This can cause control chattering and waste external electrical energy, and to date the chattering problem in the method has not been solved.[12,15] The second object of this work is, therefore, to develop a method to prevent undesirable control chattering and further reduce external energy consumption. The method to prevent control chattering is expected to be effective for not only hybrid vibration suppression, but also energy-recycling semiactive vibration suppression. In developing this method, the authors focused on energy flow between mechanical and electrical energy. The proposed method works by monitoring the voltage of the piezoelectric transducer related to the stored electrical energy, unlike a simple method, such as putting limit values on parameters (e.g., amplitudes of vibrations). It avoids a trial-and-error process to determine the limit values and can respond to complicated vibrations, such as random excitations. The effectiveness of the chattering prevention method in saving external energy was proven in experiments.

II. Equations of Motion of MDOF Systems with Piezoelectric Transducers

To investigate vibration suppression that is applicable to MDOF systems with multiple piezoelectric transducers, equations of motion

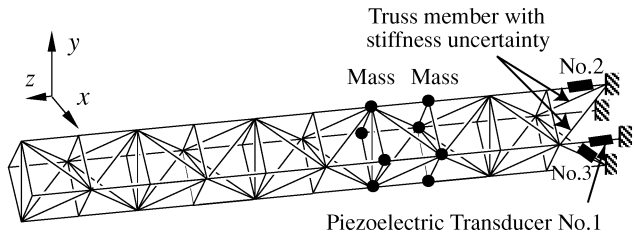

Fig. 1 Ten-bay truss structure with three piezoelectric transducers.

for an MDOF system are first derived. Let us consider a 10-bay truss structure having n_p piezoelectric transducers as shown in Fig. 1. The relation between tensile load f_p, elongation u_p, voltage V_p, and electric charge Q of the piezoelectric transducer can be written as

$$f_p = k_p u_p - b_p Q \tag{1}$$

$$V_p = -b_p u_p + Q \big/ C_p^S \tag{2}$$

where k_p, b_p, and C_p^S are functions of the characteristics of the piezoelectric transducer.[16] When local dynamics in piezoelectric transducers are negligible, by using Eqs. (1) and (2) the equation of motion for a truss structure with multiple piezoelectric transducers is

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{B}_p\boldsymbol{Q} + \boldsymbol{w} \tag{3}$$

Expressing Eq. (2) in vector-matrix form, we have

$$\boldsymbol{V}_p = -\boldsymbol{B}_p^T\boldsymbol{x} + \boldsymbol{C}_p^{-1}\boldsymbol{Q} \tag{4}$$

Introducing the damping ratio ζ for all vibration modes, the equation of motion in modal coordinates is expressed as

$$\ddot{\boldsymbol{q}} + 2\boldsymbol{\Xi}\boldsymbol{\Omega}^{\frac{1}{2}}\dot{\boldsymbol{q}} + \boldsymbol{\Omega}\boldsymbol{q} - \boldsymbol{\Phi}^T\boldsymbol{B}_p\boldsymbol{Q} - \boldsymbol{\Phi}^T\boldsymbol{w} = 0 \tag{5}$$

where

$$\boldsymbol{\Phi} \equiv [\phi_1, \phi_2, \ldots, \phi_n] \tag{6}$$

$$\boldsymbol{\Omega} \equiv \text{diagonal}\left[\omega_i^2\right] \tag{7}$$

$$\boldsymbol{\Xi} \equiv \text{diagonal}[\zeta] \tag{8}$$

Here, ω_i and ϕ_i are obtained by solving the eigenvalue problem for the homogeneous part of Eq. (3).

III. Control Strategy for Hybrid Vibration Suppression

A. Linear-Quadratic-Regulator Active Control Theory

The modal equation (5) can be rewritten as

$$\dot{\boldsymbol{z}} = \boldsymbol{A}\boldsymbol{z} + \boldsymbol{B}\boldsymbol{Q} + \boldsymbol{D}\boldsymbol{w} \tag{9}$$

where

$$\boldsymbol{z} \equiv [\boldsymbol{q}^T, \dot{\boldsymbol{q}}^T]^T \tag{10}$$

$$\boldsymbol{A} \equiv \begin{bmatrix} 0 & \boldsymbol{I} \\ -\boldsymbol{\Omega} & -2\boldsymbol{\Xi}\boldsymbol{\Omega}^{\frac{1}{2}} \end{bmatrix}, \qquad \boldsymbol{B} \equiv \begin{bmatrix} 0 \\ \boldsymbol{\Phi}^T\boldsymbol{B}_p \end{bmatrix}, \qquad \boldsymbol{D} \equiv \begin{bmatrix} 0 \\ \boldsymbol{\Phi}^T \end{bmatrix} \tag{11}$$

and $\boldsymbol{I}$ is the unit matrix. Equation (9) indicates that well-developed linear control theories could be applied, if the value of $\boldsymbol{Q}$ of the system is directly controlled. In linear-quadratic-regulator (LQR) control theory,[17] the optimal linear control that minimizes

$$J \equiv \int_0^\infty \left(\boldsymbol{z}^T\boldsymbol{W}_1\boldsymbol{z} + \boldsymbol{Q}^T\boldsymbol{W}_2\boldsymbol{Q}\right) \mathrm{d}t \tag{12}$$

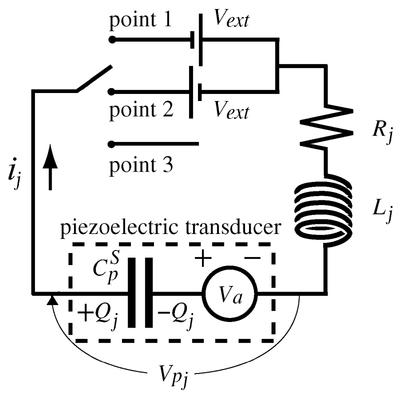

Fig. 2 Electric circuit C for hybrid vibration suppression connected to *j*th piezoelectric transducer.

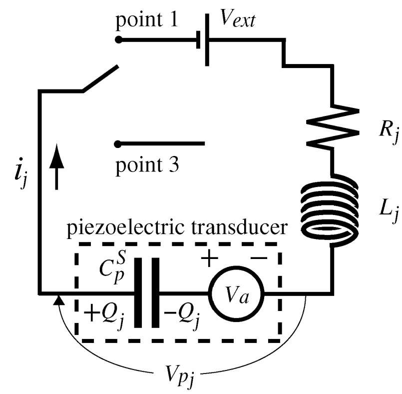

Fig. 3 Electric circuit C_1 to derive control logic of point 1 connected to *j*th piezoelectric transducer.

is to control $\boldsymbol{Q}$ as

$$\boldsymbol{Q} = \boldsymbol{Q}_T \equiv -\boldsymbol{F}\boldsymbol{z} \tag{13}$$

and the gain matrix $\boldsymbol{F}$ is given by

$$\boldsymbol{F} \equiv \boldsymbol{W}_2^{-1}\boldsymbol{B}^T\boldsymbol{P} \tag{14}$$

Here, $\boldsymbol{P}$ is a positive-definite solution of the following Riccati equation:

$$\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{W}_2^{-1}\boldsymbol{B}^T\boldsymbol{P} + \boldsymbol{W}_1 = 0 \tag{15}$$

B. Control Strategy

Hybrid vibration suppression is implemented with an electric circuit C composed of a switch, an inductor, a resistor, and two external voltage suppliers, as shown in Fig. 2. The external suppliers are assumed to have a constant positive voltage of $V_{\text{ext}}(>0)$ and are placed in parallel with alternate directions. The jth piezoelectric transducer is connected to the jth electric circuit having an inductor L_j and a resistor R_j. The jth piezoelectric transducer has electric charge Q_j and voltage V_{pj}. V_a in the figure indicates a voltage generator related to the piezoelectric effect that is expressed in the first term of the right-hand side in Eq. (4). For a system with the jth electric circuit C, it is clear that

1) when the switch is connected to point 1:

$$L_j\ddot{Q}_j + R_j\dot{Q}_j + V_{pj} = -V_{\text{ext}} \tag{16}$$

2) when the switch is connected to point 2:

$$L_j\ddot{Q}_j + R_j\dot{Q}_j + V_{pj} = V_{\text{ext}} \tag{17}$$

3) when the switch is connected to point 3:

$$\dot{Q}_j = 0 \tag{18}$$

By substituting the jth row in Eq. (4), Eq. (16) can be rewritten as

$$L_j\ddot{Q}_j + R_j\dot{Q}_j + Q_j/C_{pj} = \left[\boldsymbol{B}_p^T\boldsymbol{x}\right]_j - V_{\text{ext}} \tag{19}$$

where $[\,]_j$ indicates the jth row.

If an active method were to be used, namely, if the value of Q_j stored in the jth piezoelectric transducer can be controlled as desired, the vibration can be suppressed by supplying an electric charge Q_{Tj} to the jth piezoelectric transducer, where Q_{Tj} is the jth element of $\boldsymbol{Q}_T$ in Eq. (13). In the active system, a larger absolute value of the supplied charge Q_j should suppress the vibration more. On the other hand, the aim of the hybrid method is to suppress vibration only by controlling the switch in the electric circuit C, instead of controlling Q_j directly. Consequently, it is necessary to determine a control strategy for the hybrid method, following the conclusions drawn by Onoda et al.[18]; that is, the switch of the jth electric circuit is controlled so that Q_j has the same polarity as Q_{Tj} and the absolute value of Q_j is as large as possible. In the hybrid method, Q_{Tj} is called the target charge.

C. Hybrid Vibration Suppression

First, let us consider the electric circuit shown in Fig. 3. This reduced electric circuit C_1 will be used for deriving a control logic of point 1 in C. It is assumed that the oscillations in the electric circuit are much faster than structural vibration. When $Q_{Tj} < 0$, Q_j should be as small (i.e., negative) as possible in view of the control strategy. For smaller Q_j, the condition $\dot{Q}_j < 0$ is more preferable than $\dot{Q}_j \geq 0$. On the one hand, when $V_{pj} \leq -V_{\text{ext}}$, Q_j will increase, if the switch is connected to point 1 in C_1, as Eq. (16) indicates. Therefore, when $Q_{Tj} < 0$ and $V_{pj} < -V_{\text{ext}}$, let us connect the switch to point 3 so that the electric charge does not increase. On the other hand, when $V_{pj} > -V_{\text{ext}}$, according to Eq. (16), Q_j will decrease, after the switch is connected to point 1. However, because the electrical oscillation follows Eq. (19), Q_j starts to increase at the peak of the electrical oscillation. Therefore, when $Q_{Tj} < 0$ and $V_{pj} > -V_{\text{ext}}$, the switch should be connected to point 1 until $\dot{Q}_j$ becomes positive, and then it should be connected to point 3. Similarly, when $Q_{Tj} \geq 0$ and $V_{pj} < -V_{\text{ext}}$, the switch should be connected to point 1 until the polarity of $\dot{Q}_j$ changes from positive to negative, and when $Q_{Tj} > 0$ and $V_{pj} \geq -V_{\text{ext}}$ the switch should be connected to point 3. In conclusion, for hybrid vibration suppression, the control logic of the switch in the jth electric circuit C_1 is expressed as follows (control logic 1):

When $Q_{Tj} < 0$
- if $V_{pj} \leq -V_{\text{ext}}$, connect to point 3,
- if $V_{pj} > -V_{\text{ext}}$, connect to point 1 until $\dot{Q}_j$ becomes positive, and to point 3;

When $Q_{Tj} \geq 0$
- if $V_{pj} < -V_{\text{ext}}$, connect to point 1 until $\dot{Q}_j$ becomes negative, and to point 3,
- if $V_{pj} \geq -V_{\text{ext}}$, connect to point 3.

Next, an electric circuit C_2 composed of points 2 and 3 is considered, to derive a control logic of point 2 in C. In the same manner as with C_1, the control logic of the switch in the jth electric circuit C_2 is expressed as follows (control logic 2):

When $Q_{Tj} < 0$
- if $V_{pj} \leq V_{\text{ext}}$, connect to point 3,
- if $V_{pj} > V_{\text{ext}}$, connect to point 2 until $\dot{Q}_j$ becomes positive, and to point 3;

When $Q_{Tj} \geq 0$
- if $V_{pj} < V_{\text{ext}}$, connect to point 2 until $\dot{Q}_j$ becomes negative, and to point 3,
- if $V_{pj} \geq V_{\text{ext}}$, connect to point 3.

Finally, the electric circuit C shown in Fig. 2 is considered again, and derive its control logic as a combination of control logics 1 and 2. Connecting the switch to point 1 or 2 so that electric current flows in the desired direction is more effective in suppressing vibration than connecting the switch to point 3. Most important, for a larger absolute value of Q_j, the electric current (i.e., $i_j = -\dot{Q}_j$) should

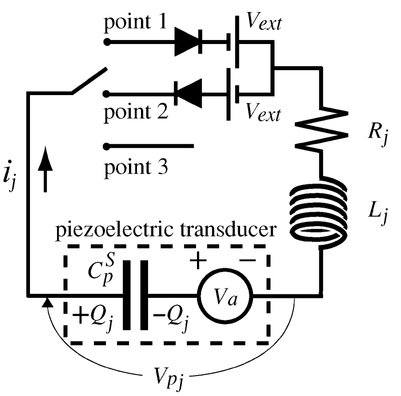

Fig. 4 Electric circuit D for refined hybrid vibration suppression connected to *j*th piezoelectric transducer.

have the same direction as the external voltage V_{ext} through which the electric current flows. Consequently, the combined control logic of the switch in the jth electric circuit C is derived as follows (control logic 3):

When $Q_{Tj} < 0$
 if $V_{pj} \leq -V_{\text{ext}}$, connect to point 3,
 if $V_{pj} > -V_{\text{ext}}$, connect to point 1 until $\dot{Q}_j$ becomes positive, and to point 3;
When $Q_{Tj} \geq 0$
 if $V_{pj} < V_{\text{ext}}$, connect to point 2 until $\dot{Q}_j$ becomes negative, and to point 3;
 if $V_{pj} \geq V_{\text{ext}}$, connect to point 3.

D. Refinement

When the electric circuit D shown in Fig. 4 is used instead of C, the hybrid vibration suppression can be implemented more easily. Because the two diodes in D prevent electric current from flowing in the undesired direction, the switch in D does not need to be controlled quickly when the current direction (i.e., the polarity of $\dot{Q}_j$) changes. A refined control logic of the switch in the jth electric circuit D can thus be written as follows (control logic 4):

When $Q_{Tj} < 0$,

connect to point 1

When $Q_{Tj} \geq 0$,

connect to point 2 (20)

Although point 3 in Fig. 4 is not required for this control logic, it will be needed for the more advanced control logic that will be discussed later. This hybrid method makes reference to the target charge $\boldsymbol{Q}_T$ obtained from the active control scheme. Advanced active controls provide target charges corresponding to multiple-mode vibrations, the number of which is greater than the number of actuators. Thus, when such a target charge is used for hybrid vibration suppression, it is possible to suppress multiple-mode vibrations having a number greater than that of piezoelectric transducers.

The bang–bang active method uses control logic 4 and the electric circuit D without an inductor and a resistor, whereas the energy-recycling semiactive method uses control logic 4 and the electric circuit D without voltage suppliers (i.e., $V_{\text{ext}} = 0$). In this sense, this investigation into the hybrid method is a comprehensive study that includes previous researches on the bang–bang active method and the energy-recycling semiactive method. Because the hybrid method and the energy-recycling semiactive method can share the same circuit, one application of the hybrid method is as an auxiliary vibration suppression for times when the vibration is too large for the semiactive method alone to suppress.

IV. Novel Method to Prevent Control Chattering

In general bang–bang vibration controls, when the vibration is suppressed enough a high-frequency change of control input usually occurs. This rapid change of control input is usually termed control chattering. In practical engineering systems, the control chattering can lead to a waste of supplied energy and an excitation of high-frequency dynamics. The control chattering also occurs in vibration suppression simulations and experiments using the energy-recycling semiactive method.[12,15] With the semiactive method, electric charge regarded as control input is generated as a result of the piezoelectric effect and stored in the piezoelectric transducers, and it increases with each switching. Even after the vibration has been suppressed sufficiently, the transducer stores a large amount of electric charge, and consequently the semiactive system usually experiences control chattering. Until now, this undesirable chattering problem has not been overcome in the method. Because the hybrid method is a combination of the bang–bang active method and the energy-recycling semiactive method, it can be affected by control chattering, which would waste the actively supplied energy. One of the simplest ideas for avoiding control chattering is to put a limit on the amplitude of vibration. For example, when maximum displacement reaches a certain value, the vibration control ceases. However, this idea is far too primitive to apply to hybrid vibration suppression because control chattering does not result directly from the amplitude of vibration, but from the ratio of the control input to the amplitude of vibration. Furthermore, this ratio depends on the vibration history that the system has gone through because the electric charge regarded as control input increases with each switching. In other words, it is impossible to determine, beforehand, a limit value for the amplitude of vibration, or the ratio at which control chattering occurs. Therefore, a more sophisticated method is needed to prevent control chattering in hybrid vibration suppression.

We focused on the energy flow between the mechanical energy of the vibrating structure and the electrical energy stored in the piezoelectric transducer, in order to establish a method to prevent control chattering. While current does not flow in an electric circuit (i.e., the switch is open), the piezoelectric transducer exchanges mechanical and electrical energy. To suppress vibration, the mechanical energy should be decreased and the electrical energy increased. However, if the electrical energy decreases while the switch is open, the decrement of the electrical energy can be converted into the mechanical energy, which would increase the structural vibration. One interpretation of this situation is that the electric charge is too large compared with the amplitude of vibration. Thus, if the hybrid vibration suppression continues, an almost still structure will be driven by the large electric charge. Because the driven structural motion changes the polarity of the target charge, the switch is closed, and electric current flows. Consequently, the electric charge takes on a large value with the opposite polarity. The structure is then forced to move in the opposite direction by the opposite electric charge. Consequently, the target charge fluctuates around the zero value, and control chattering occurs. The conclusion is that control chattering can occur when the electrical energy stored in the piezoelectric transducer decreases while the switch is open. Therefore, to prevent control chattering, the hybrid vibration suppression should be paused when the switch is open and the electrical energy stored decreases.

After the pause duration is determined, we are faced with two alternatives. The first alternative is to discharge the stored electric charge through a resistor and to pause the hybrid vibration suppression. Although this can remove the root of the chattering problem, the electrical energy stored for recycling disappears. The second alternative is to pause the hybrid vibration suppression and keep the electrical energy in the piezoelectric transducer (i.e., the electric current is not allowed to flow). The stored charge can be used to suppress the next vibration that has a larger amplitude, although it can shift the neutral position of the structure. The second alternative is adopted, although both methods can be combined in an actual design employing the hybrid method. Furthermore, the hybrid method should resume its operation to suppress an increasing vibration when electrical energy reaches the value it had at the last switching.

A. Energy of Structures with Piezoelectric Transducers

Here, the electrical energy of an internal system is derived for the control logic of the chattering prevention method. The internal system is assumed to comprise the structure, the piezoelectric transducer, and the electric circuit except for the voltage suppliers, while the voltage suppliers belong to the external system. The integral of the product of $\dot{\boldsymbol{x}}$ and $\boldsymbol{w}$ with respect to time t from 0 to t'

$$W_w \equiv \int_0^{t'} \dot{\boldsymbol{x}}^T \boldsymbol{w}\, \mathrm{d}t \tag{21}$$

represents the work done by the external force. To express the equation of electricity in vector-matrix form, keeping in mind that $1 \le i \le n_p$, the scalar equations (17) are assembled to form a vector equation as

$$\boldsymbol{L}\ddot{\boldsymbol{Q}} + \boldsymbol{R}\dot{\boldsymbol{Q}} + \boldsymbol{V}_p = \boldsymbol{V}_{\text{ext}} \tag{22}$$

The work done by the external voltage is expressed as

$$W_V \equiv \int_0^{t'} \dot{\boldsymbol{Q}}^T \boldsymbol{V}_{\text{ext}}\, \mathrm{d}t \tag{23}$$

From Eqs. (3), (4), and (21–23), we obtain

$$W_w + W_V = \left[\frac{1}{2}\dot{\boldsymbol{x}}^T\boldsymbol{M}\dot{\boldsymbol{x}} + \frac{1}{2}\boldsymbol{x}^T\left(\boldsymbol{K} - \boldsymbol{B}_p\boldsymbol{C}_p\boldsymbol{B}_p^T\right)\boldsymbol{x}\right]_0^{t'} + \left[\frac{1}{2}\dot{\boldsymbol{Q}}^T\boldsymbol{L}\dot{\boldsymbol{Q}} + \frac{1}{2}\boldsymbol{V}_p^T\boldsymbol{C}_p\boldsymbol{V}_p\right]_0^{t'} + \int_0^{t'} \dot{\boldsymbol{Q}}^T\boldsymbol{R}\dot{\boldsymbol{Q}}\, \mathrm{d}t \tag{24}$$

Because the first term on the right-hand side is independent of the increment path of time and is composed of only mechanical variables, the mechanical energy is given by

$$\tfrac{1}{2}\dot{\boldsymbol{x}}^T\boldsymbol{M}\dot{\boldsymbol{x}} + \tfrac{1}{2}\boldsymbol{x}^T\left(\boldsymbol{K} - \boldsymbol{B}_p\boldsymbol{C}_p\boldsymbol{B}_p^T\right)\boldsymbol{x} \tag{25}$$

Note that the mechanical energy is expressed with the constant-voltage stiffness matrix of the truss structure $(\boldsymbol{K} - \boldsymbol{B}_p\boldsymbol{C}_p\boldsymbol{B}_p^T)$, instead of the constant-charge stiffness matrix $\boldsymbol{K}$. Because the second term on the right-hand side is composed of electrical variables, electrical energy can be written as

$$\tfrac{1}{2}\dot{\boldsymbol{Q}}^T\boldsymbol{L}\dot{\boldsymbol{Q}} + \tfrac{1}{2}\boldsymbol{V}_p^T\boldsymbol{C}_p\boldsymbol{V}_p \tag{26}$$

This definition of electrical energy will be used to prevent control chattering.

B. Control Logic to Prevent Control Chattering

As described earlier, to derive a control logic for preventing control chattering, the transition of electrical energy while the switch is open (i.e., $\dot{\boldsymbol{Q}} = 0$) is analyzed. According to the definition in Eq. (26), the electrical energy stored in the jth piezoelectric transducer is $C_{pj}V_j^2/2$. Let $V_{\text{ref},j}$ be the voltage across the jth piezoelectric transducer immediately after the current stops flowing and V_j be the voltage. $V_{\text{ref},j}$ is referred to as the reference voltage of the jth transducer. It should be remembered that the hybrid vibration suppression is to be paused when the switch is open and the electrical energy stored in the piezoelectric transducer decreases. Based on control logic 4, a control logic for the switch in the jth electric circuit D is derived. The advanced control logic for hybrid vibration suppression with chattering prevention is written as follows (control logic 5):

When $Q_{Tj} < 0$
  if $|V_j| \ge |V_{\text{ref},j}|$, connect to point 1 until $Q_{Tj} \ge 0$,
  if $|V_j| < |V_{\text{ref},j}|$, connect to point 3;
When $Q_{Tj} \ge 0$
  if $|V_j| \ge |V_{\text{ref},j}|$, connect to point 2 until $Q_{Tj} < 0$,
  if $|V_j| < |V_{\text{ref},j}|$, connect to point 3.

This prevention method is simple enough to be implemented in actual systems. In actual systems, even if the electric current does not flow, the electric charge in the piezoelectric transducer will sometimes be gradually discharged as a result of inner leakage or consumption by measuring devices. Given such a discharge, the absolute value of the reference voltage in control logic 5 should be decreased gradually with time at the discharge rate. Because measuring the flow of electric current is impractical, $V_{\text{ref},j}$ for control logic 5 can be substituted with the most recent voltage measured after current stops flowing.

V. Numerical Simulations

Numerical simulations using the 10-bay truss structure shown in Fig. 1 were carried out to compare the performance of the hybrid method with those of the energy-recycling semiactive method and the bang–bang active method. This 10-bay truss structure design was also used in the experiments discussed later. Each axial member had a stiffness per unit length of 1.99×10^6 N and a length of 0.38 m. Each diagonal member had a length of $\sqrt{2}$ times that of the axial member and the same stiffness per length as the axial member. The axial member weighted 35.7 g, and the diagonal member weighted 46.3 g. Each node weighted 67.9 g. The values of k_p, b_p, and C_p^S in the mathematical model of the piezoelectric transducer expressed in Eqs. (1) and (2) were $k_p = 5.75 \times 10^6$ N/m, $b_p = 2.57 \times 10^5$ N/C, and $C_p^S = 1.17 \times 10^{-5}$ F. The inductance and resistance were set as $L = 2.23 \times 10^{-3}$ H and $R = 1.0\ \Omega$. The damping ratio ζ of each mode was assumed to be 0.36%. These values were based on the experimental system parameters. The following numerical simulation had a time step of 1.0×10^{-7} s, which was small enough compared with the highest mechanical vibration mode (whose period was 4.9×10^{-4} s) and the electrical oscillation (whose period was 1.0×10^{-3} s). Because the numerical simulations were intended to reveal the relative performances of different vibration suppressions, the control chattering preventive was not considered, and systems were implemented with the electric circuit D and control logic 4.

A. Hybrid Vibration Suppression with a Single Transducer

To understand how the hybrid method worked, vibration suppression for a truss structure having a single piezoelectric transducer was first simulated. The initial modal velocity of the first mode was set to -0.01 mkg$^{1/2}$/s, and the initial modal displacements and velocities of all of the other modes were set to 0. The subsequent vibrations were suppressed. Here, the state vector was assumed to be known, and an observer was not used. To derive the value of Q_T from the LQR control scheme, the lowest mode was assumed to be controlled, and the weighting matrix $\boldsymbol{W}_1$ in Eq. (12) was set to

$$\boldsymbol{W}_1 = \text{diagonal}\left[1, 1\big/\omega_1^2\right] \tag{27}$$

and $\boldsymbol{W}_2$ was set as a scalar because the system had one transducer. $\boldsymbol{W}_2$ was approximately optimized as 1.0×10^1 kg, with which I_{rms} was minimal. The following integral was calculated as the performance measure:

$$I_{\text{rms}} \equiv \int_{t_S}^{t_E} \delta_{\text{rms}}\, \mathrm{d}t \tag{28}$$

where $t_S = 0.0$ s and $t_E = 0.5$ s. Uncontrolled (or residual) modes can be excited when controlled (or target) modes are suppressed by a method whose control input changes sharply, as is the case with the bang–bang active method, the energy-recycling semiactive method, and the hybrid method. I_{rms} is not only a measure of vibration suppression, but it also is a measure of the influence of uncontrolled modes.

Figure 5 plots I_{rms} for the hybrid system given $V_{\text{ext}} = 0.4$ or 0.8 V. When the resistance is smaller than 5 Ω, the vibration suppression at both voltages is fairly good. However, it deteriorates when the resistance exceeds 5 Ω. This tendency means that a large resistor dissipates the electrical energy stored in the piezoelectric transducer, which causes the suppression performance to deteriorate. The hybrid system having a supply voltage of 0.8 V better suppresses vibration than the system with 0.4-V supply. For comparison, I_{rms} for a system with the energy-recycling semiactive method and a system with the bang–bang active method are also plotted in the figure. The performance of the bang–bang active method is independent of the value of resistance. The hybrid method was more effective than the semiactive method. Figure 5 also shows that when the resistance is

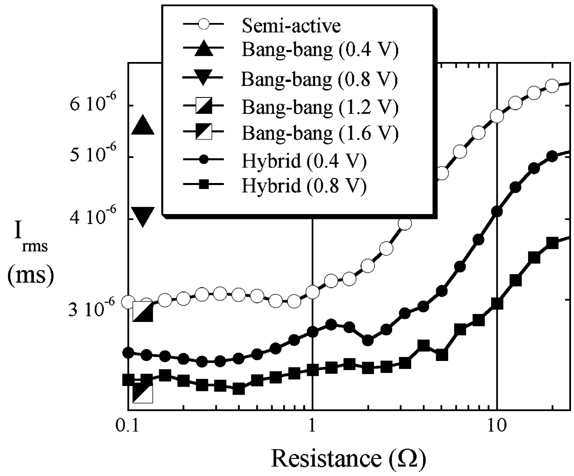

Fig. 5 Performance measure I_{rms} for single-mode vibration suppression as a function of resistance.

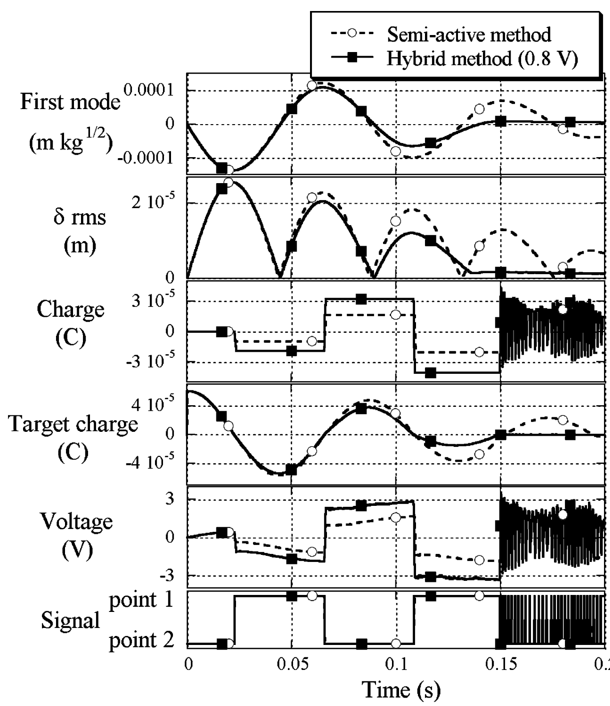

Fig. 6 Time histories of single-mode vibration suppression (hybrid and semiactive methods).

1.0 Ω the hybrid system with a 0.4-V supply voltage has almost the same performance as the bang–bang active system with a 1.2-V supply voltage. The ratio of the two voltages is nearly 3. This difference is caused by the hybrid vibration suppression mechanism. Because these control methods change their control inputs instantaneously, they cause some additional dynamics of the MDOF systems. This explains the unsmoothed shape of curves in Fig. 5 (and also later curves).

Figure 6 shows the time histories of vibration suppression for the hybrid method with $V_{\text{ext}} = 0.8$ V and the energy-recycling semiactive method. Here $R = 1.0\ \Omega$. The target charge is calculated from the value of the first mode. In both systems, the switching happens around the peak of the first modal displacement. By switching between points 1 and 2, the polarity of electric charge stored in the piezoelectric transducer is equal to that of the target charge, as intended. Both electric charge and voltage alternately take positive and negative values. The hybrid method has a larger change in electric charge compared with the semiactive method, which is caused by the active voltage supply in D. Consequently, the hybrid method

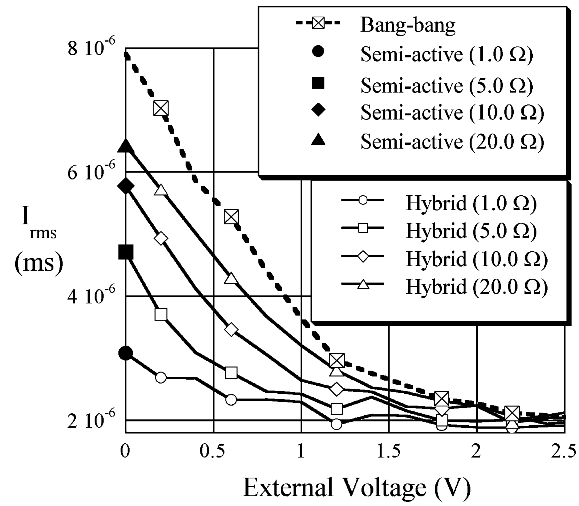

Fig. 7 Performance measure I_{rms} for single-mode vibration suppression as a function of external voltage.

has a large absolute value of the electric charge, and it suppresses the vibration more quickly than the semiactive method does. The hybrid method not only recycles the mechanical energy, but also prevents the externally supplied energy from being dissipating so that it can be used later on for vibration suppression. One conclusion drawn from the comparison with the bang–bang active method is that the hybrid system saves a significant amount of external energy. The figure shows that control chattering occurs after the vibration has been mostly suppressed ($t = 0.15$ s).

Figure 7 plots I_{rms} for the three systems as a function of external voltage, under the same conditions as in Fig. 5. This figure indicates that the vibration suppression capability of the hybrid and bang–bang active systems depends on the external voltage. When the external voltage is small, the hybrid system's performance is similar to that of the semiactive system, but when the external voltage is larger than 2 V its performance is similar to the bang–bang active system's.

In summary, the numerical simulations confirmed that the hybrid method outperforms the energy-recycling semiactive method and the bang–bang active method in suppressing single-mode vibrations.

B. Multiple-Mode Vibration Suppression with Multiple Transducers

To see whether the hybrid method were effective in suppressing multiple-mode vibrations of a system having multiple piezoelectric transducers, vibration suppression of a 10-bay truss structure with three piezoelectric transducers was simulated. For simplicity, the three transducers were assumed to be identical. An initial velocity of (0.1, 0.1, 0.1) m/s in x–y–z coordinates was assumed for the resting structure, and the subsequent free vibration was suppressed. The lowest six modes were assumed to be controlled in the LQR control scheme to obtain $\boldsymbol{Q}_T$. The weighting matrices of Eq. (12) were set to

$$\boldsymbol{W}_1 = \text{diagonal}\left[1, \ldots, 1, 1/\omega_1^2, \ldots, 1/\omega_6^2\right] \tag{29}$$

$$\boldsymbol{W}_2 = 1.0 \times 10^{-1}\text{diagonal}[1, 1, 1]\ \text{kg} \tag{30}$$

based on the approximately optimal value of $\boldsymbol{W}_2$ for the truss structure having three piezoelectric transducers. Figure 8 plots I_{rms} for this truss structure as a function of resistance for the hybrid system with $V_{\text{ext}} = 0.4$ or 0.8 V, the semiactive system and the bang–bang active system with $V_{\text{ext}} = 0.4$, 0.8, 1.2 or 1.6 V. The performance measure of vibration suppression was I_{rms} expressed by Eq. (28) with t_S set to 0.0 s and t_E to 0.5 s. Similar to the single piezoelectric transducer case, the hybrid method is more effective than the semiactive method in suppressing the vibration of an MDOF structure with multiple piezoelectric transducers. When the resistance is 0.6 Ω, the hybrid system with a 0.8-V supply has almost the same

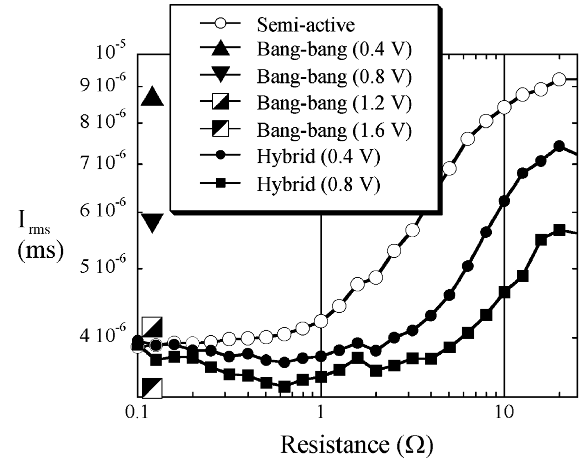

Fig. 8 Performance measure I_{rms} for multiple-mode vibration suppression with three piezoelectric transducers.

performance as the bang–bang active system with a 1.6-V supply. The hybrid system saves more external energy compared with the bang–bang active method, and suppresses vibration better than the semiactive method.

C. Robustness of Suppression

Because the energy-recycling semiactive method does not supply any external energy, there is no risk of the system instability, such as divergence. On the other hand, the bang–bang active method supplies external energy and thus raises a possibility of system instability. Therefore, the bang–bang active method is thought to have limited robustness to model errors. In actual systems, vibration modes can not be measured directly and must be estimated by an observer (e.g., Kalman filter[17]). The observation spillover caused by model errors or noise usually leads to a deterioration in vibration suppression performance. Therefore, numerical simulations using an observer with intentional model errors were conducted. An initial velocity of (0.1, 0.0, 0.0) m/s was given to the resiting structure, and the subsequent free vibration was suppressed by using the hybrid method with $V_{\mathrm{ext}} = 0.4$ or 0.8 V, the energy-recycling semiactive method, and the bang–bang active method with $V_{\mathrm{ext}} = 0.4$, 0.8, or 1.2 V. Because the locations of the transducers were not symmetrical to the x–z plane, the impulsive force excited not only the modes symmetrical with respect to the plane but also the others. The observation sensor was assumed to measure the x-direction displacement of the tip node, and the lowest six modes were suppressed using three piezoelectric transducers. To obtain $\boldsymbol{Q}_T$, the lowest six modes were assumed to be controlled in the LQR control scheme, and the weighting matrices of Eq. (12) were set to

$$\boldsymbol{W}_1 = \mathrm{diagonal}\left[1, \ldots, 1, 1\big/\omega_1^2, \ldots, 1\big/\omega_6^2\right] \tag{31}$$

$$\boldsymbol{W}_2 = w_2 \times \mathrm{diagonal}[1, 1, 1]\ \mathrm{kg} \tag{32}$$

where w_2 is a variable in this LQR control design. It is impractical for an actual observer to measure the electric charge stored in the piezoelectric transducer. However, because to measure voltage is a realistic possibility, an observer in terms of voltage can be constructed. Substituting Eq. (4) into Eq. (9), we get

$$\dot{\boldsymbol{z}} = \boldsymbol{A}'\boldsymbol{z} + \boldsymbol{B}'\boldsymbol{V}_p + \boldsymbol{D}\boldsymbol{w} \tag{33}$$

in terms of matrices defined as

$$\boldsymbol{A}' \equiv \begin{bmatrix} 0 & \boldsymbol{I} \\ -\boldsymbol{\Omega} + \boldsymbol{\Phi}^T\boldsymbol{B}_p\boldsymbol{C}_p\boldsymbol{B}_p^T\boldsymbol{\Phi} & -2\boldsymbol{\Xi}\boldsymbol{\Omega}^{\frac{1}{2}} \end{bmatrix}, \qquad \boldsymbol{B}' \equiv \begin{bmatrix} 0 \\ \boldsymbol{\Phi}^T\boldsymbol{B}_p\boldsymbol{C}_p \end{bmatrix} \tag{34}$$

Let $\boldsymbol{y}$ be a vector composed of sensor outputs. This vector is assumed to be a linear function of the state vector $\boldsymbol{z}$, that is,

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{z} \tag{35}$$

where $\boldsymbol{C}$ is the output matrix. An observer (Kalman filter[17]) for estimating $\boldsymbol{z}$ can be written as

$$\dot{\hat{\boldsymbol{z}}} = \boldsymbol{A}'\hat{\boldsymbol{z}} + \boldsymbol{B}'\boldsymbol{V}_p + \boldsymbol{\Gamma}(\boldsymbol{y} - \boldsymbol{C}\hat{\boldsymbol{z}}) \tag{36}$$

where the observer gain matrix $\boldsymbol{\Gamma}$ is defined as

$$\boldsymbol{\Gamma} \equiv \boldsymbol{P}\boldsymbol{C}^T\boldsymbol{V}_2^{-1} \tag{37}$$

Here, $\boldsymbol{P}$ is a positive-definite solution of

$$\boldsymbol{A}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}^T - \boldsymbol{P}\boldsymbol{C}^T\boldsymbol{V}_2^{-1}\boldsymbol{C}\boldsymbol{P} + \boldsymbol{V}_1 = 0 \tag{38}$$

For simplicity, it is assumed that

$$\boldsymbol{V}_1 = v_1\begin{bmatrix} 0 & 0 \\ 0 & \boldsymbol{I}_6 \end{bmatrix}, \qquad \boldsymbol{V}_2 = \boldsymbol{I}_3 \tag{39}$$

where v_1 is a variable in this observer design.

For vibration suppression, $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ are design parameters for the target charge $\boldsymbol{Q}_T$, whereas $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ are design parameters for estimating $\boldsymbol{z}$. Therefore, the control system depends on two scalar values of w_2 and v_1. First, an optimal parameter combination of w_2 and v_1 for which I_{rms} value is minimal is determined for each system. By performing an extensive parametric study, we found that the energy-recycling semiactive system's optimal parameter combination was $w_2 = 1.0 \times 10^0$ kg and $v_1 = 1.0 \times 10^8$ m^{2}kg/s^2, the hybrid system's with $V_{\mathrm{ext}} = 0.4$ V was $w_2 = 1.0 \times 10^{-1}$ kg and $v_1 = 3.2 \times 10^7$ m^{2}kg/s^2, and the hybrid system's with $V_{\mathrm{ext}} = 0.8$ V was $w_2 = 1.0 \times 10^{-1}$ kg and $v_1 = 1.0 \times 10^7$ m^{2}kg/s^2. Next, after designing these three systems the stiffness EA of two truss members (shown in Fig. 1 as members with stiffness uncertainty) was varied by the factor of EA/EA_{nominal}, keeping all other parameters of the three systems unchanged.

Figure 9 plots I_{rms} as a function of EA/EA_{nominal}. The slopes of the curves for the hybrid system and the semiactive system are more gentle than those of the bang–bang active system. This means that the hybrid system's performance does not deteriorate as much because of model errors. Thus, it appears that the hybrid method is robust against model errors that would affect the bang–bang active method. Because the initial velocity of (0.1, 0.0, 0.0) m/s is assumed for all of the cases, as EA/EA_{nominal} increases, the resultant amplitude of displacements becomes small. Therefore, as EA/EA_{nominal} increases, the I_{rms} value tends to become small. That is why the plotted lines are not symmetrical with respect to $EA/EA_{\mathrm{nominal}} = 1$.

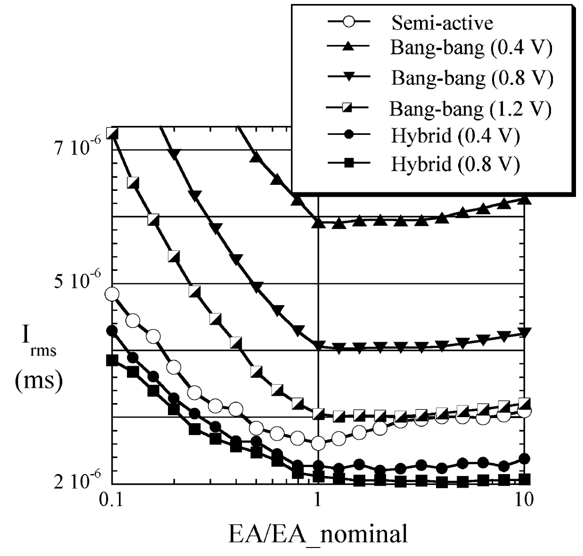

Fig. 9 Performance measure I_{rms} for robustness of vibration suppression to intentional model errors.

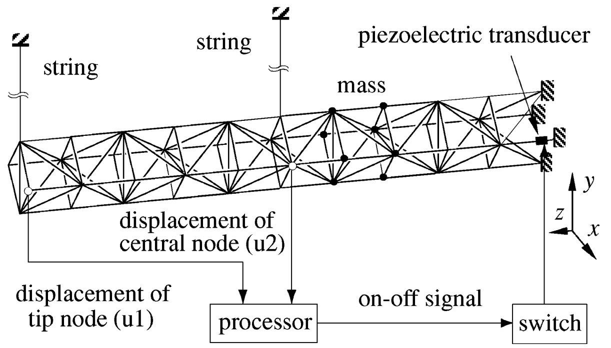

Fig. 10 Control flow of hybrid vibration suppression experiment.

VI. Experiments

The experiments used a 10-bay cantilevered truss structure shown in Fig. 10. The members at the base could not support the total weight of the structure, and so the structure was attached to strings at the tip and the central nodes (Fig. 10). Because of the strings, the motion of the truss structure was restricted to the x–z plane. Two displacement sensors were installed at the tip and the central nodes to measure x-directional displacements u_1 and u_2. In the following experiments, only horizontal bending vibrations in the x–z plane were excited and suppressed. A commercially available NEC/TOKIN ASB171C801NP0 piezoelectric transducer was used. It had a length of 0.22 m and a mass of 93.0 g and was composed of 1300 piezoceramic layers.

In the same way as in the numerical simulation, the LQR control scheme was used to obtain the target charge $\boldsymbol{Q}_T$ for suppressing the lowest two modes. The values of the weighting matrices $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ were also determined in the same way as in the numerical simulation. The control flow shown in Fig. 10 using control logics 4 and 5 was as follows. First, the measured displacements of the tip u_1 and the central u_2 nodes were sent through an analog-digital converter to a processor. Next, the processor calculated the target charge and sent, if necessary, a signal through a digital-analog converter to the switch. On receiving the signal, the switch would change its connection point. The observer expressed in Eq. (36) was used to estimate the first and second modes. The observer gain $\boldsymbol{\Gamma}$ in Eq. (36) was determined by the numerical simulation as follows. For simplicity, it was assumed that

$$\boldsymbol{V}_1 = v_1 \begin{bmatrix} 0 & 0 \\ 0 & \boldsymbol{I}_2 \end{bmatrix}, \qquad \boldsymbol{V}_2 = \boldsymbol{I}_4 \tag{40}$$

The truss structure was excited at two frequencies of the first and second vibration modes. After the excitation ceased and while the resulting free vibration lasted, the estimation error integral

$$\int_{0\,\text{sec}}^{10\,\text{sec}} |\hat{z} - z|^2 \, \mathrm{d}t \tag{41}$$

was calculated as the observed performance measure. The v_1 in Eq. (40) was determined so that the integral was minimized. The observer for the experimental setup was built with the observer gain $\boldsymbol{\Gamma}$ obtained from the value of v_1.

A. Single-Mode Vibration Suppression

The single-mode vibration suppression experiments were performed using a single piezoelectric transducer to see whether the hybrid method suppressed the vibration in an actual structure and to see if the chattering prevention method worked as designed. The first horizontal bending mode (11.52 Hz) in the x–z plane was excited by using a permanent magnet and a voice coil connected to the floor. After the excitation, the voice-coil circuit was opened so that the vibration of the truss structure would not be damped by the energy dissipating from the voice-coil circuit. The subsequent free vibration was then suppressed using the hybrid and the energy-recycling semiactive methods. Figure 11 shows the time histories

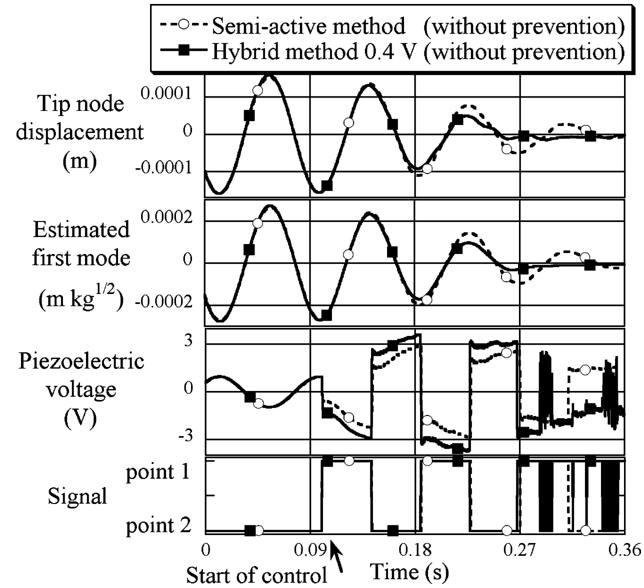

Fig. 11 Experimental results for single-mode vibration suppression without chattering prevention.

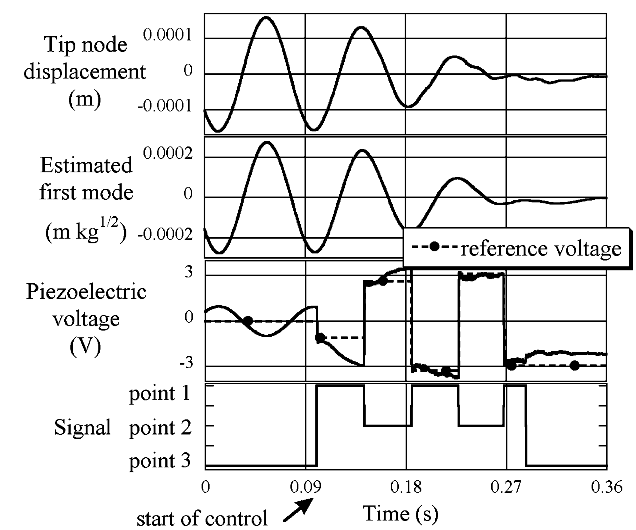

Fig. 12 Experimental results for single-mode vibration suppression using hybrid method with chattering prevention.

for the system implemented with control logic 4. Before vibration suppression, the electric circuit was open so that electric current did not flow. Vibration suppression began at $t = 0.1$ s, and the first-mode vibration was quickly suppressed in the following 0.16 s. During this time, the switch in the electric circuit D changed from point 1 to point 2 and from point 2 to point 1 around each peak of the estimated first mode, and consequently the voltage changed from negative to positive and from positive to negative. The absolute value of voltage in the hybrid method was larger than in the semiactive method. That is why the hybrid method was better in suppressing vibration than the semiactive method. This figure shows, after the vibration had been sufficiently suppressed, the hybrid method developed control chattering. Figure 12 shows the time histories for the system implemented with control logic 5. Comparing this figure with Fig. 11 clearly illustrates the effectiveness of the method to prevent control chattering. The processor compared the voltage and the reference voltage and paused the hybrid control after $t = 0.29$ s. The chattering prevention method worked as intended, and it saved energy from the external source. These experiments proved that the hybrid system is better at suppressing single-mode vibration and that the chattering prevention method is effective in stopping chattering and saving external energy.

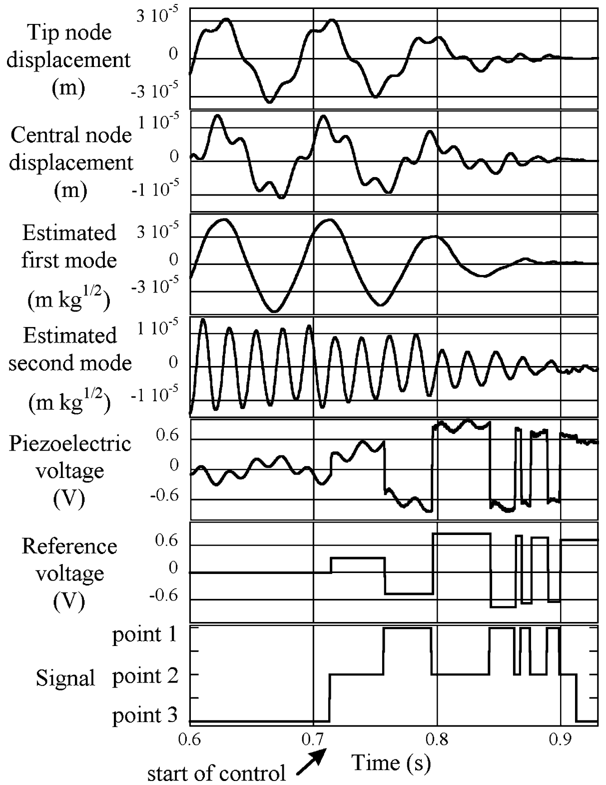

Fig. 13 Experimental results for multiple-mode vibration suppression using hybrid method with chattering prevention.

B. Multiple-Mode Vibration Suppression

Next, the multiple-mode vibration suppression experiment was performed using three piezoelectric transducers. The lowest two horizontal bending modes (11.52 and 48.00 Hz) in the x–z plane were first excited using the permanent magnet and the voice coil. After the voice-coil circuit was opened, the subsequent free vibration of the two modes was suppressed by using the hybrid method with control logic 5. The lowest two modal displacements were estimated using Eq. (36). Figure 13 shows the obtained time histories. The control started at $t = 0.71$ s, and the vibration had been sufficiently suppressed in the following 0.2 s. Until $t = 0.84$ s, the switching mainly suppressed the first mode vibration. After that, it mainly suppressed the second vibration mode. The hybrid method flexibly responded to multiple vibration modes depending on their relative amplitudes. The figure also shows that the control chattering preventive worked as designed.

C. Suppressing Vibrations Caused by Random Excitation

Experiments to suppress the vibrations caused by random excitations were performed to assess the performance of the hybrid method under more severe vibrational conditions than transient vibrations. By using an inverse fast Fourier transform, four different random-force functions were created on a computer. The power spectral density of random forces was 0.1 N^2/Hz in the range of from 10.0 to 50.0 Hz, and its value was 0 over the rest of the frequency range, which covered the first (11.52 Hz) and the second (48.00 Hz) modes. A voltage corresponding to the random-force functions was sent to the voice-coil circuit to excite the structure. The random forces were applied at the fifth bay of the truss structure in the x direction. The structure was at rest at $t = 0$ and was excited by the random force for 5 s without any vibration suppression. The random force was maintained for duration of the experiment, which tested the following four methods: the hybrid method with $V_{ext} = 0.1$ V, the energy-recycling semiactive method, the bang–bang active method with $V_{ext} = 0.1$ V, and no control. The value of

$$I_{2rms} \equiv \int_{10.0\text{ s}}^{20.0\text{ s}} \sqrt{\frac{u_1^2 + u_2^2}{2}}\,\mathrm{d}t \tag{42}$$

Table 1 I_{2rms} in experiment of suppressing vibration excited by random force (unit: $\times 10^{-4}$ ms)

Control method\input set	Set A	Set B	Set C	Set D
Hybrid method ($V_{ext} = 0.1$ V)	0.981	0.975	0.995	0.970
Energy-recycling semiactive method	1.427	1.418	1.782	1.565
Bang–bang active method ($V_{ext} = 0.1$ V)	2.674	2.565	2.835	2.513
No control	8.395	8.504	9.054	8.457

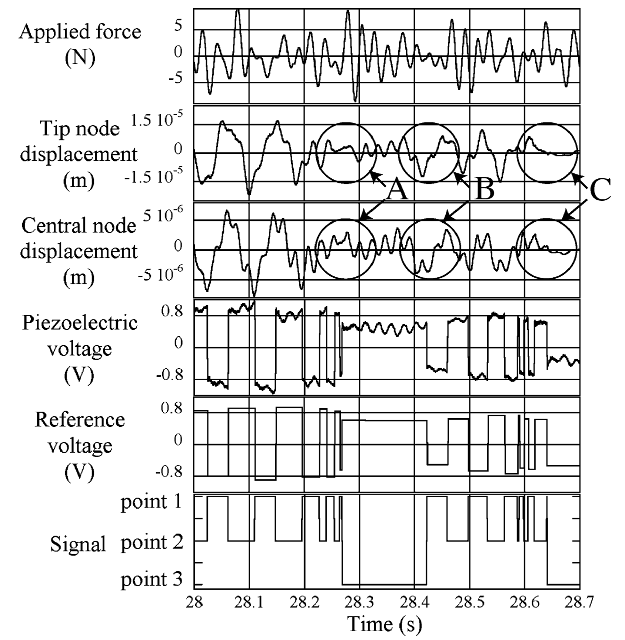

Fig. 14 Experimental results for suppressing vibration excited by random force by using hybrid method with chattering prevention.

was calculated as a measure of vibration suppression. The values of I_{2rms} for the four systems were measured for four random-force functions (from A to D), as shown in Table 1. The values of I_{2rms} for the hybrid method were smaller than those for the semiactive method and the bang–bang active methods, showing that the hybrid method was the most effective in suppressing the vibrations excited by the random forces. Figure 14 shows a time history of a system while a random force was being applied to it. When displacements of the tip u_1 and central u_2 nodes had become small enough (circles A and C), the hybrid control paused to save the electrical energy. When the displacements increased again (circle B), the hybrid control resumed. The vibration suppression experiments with random excitations confirmed that the hybrid method was better than the energy-recycling semiactive method or the bang–bang active method for a given supply voltage and that it could prevent control chattering in systems subject to actual disturbances.

VII. Conclusions

An innovative hybrid method of vibration suppression combines a bang–bang active method and an energy-recycling semiactive method. With it, some of the electrical energy needed for vibration control is transferred from mechanical vibration energy, instead of being provided entirely from external energy sources. It also reuses the externally supplied energy many times for vibration suppression, by preventing it from being dissipating. Consequently, it reduces the energy actively supplied from external sources. Control logics for the hybrid method were designed so that the hybrid method could suppress multiple-mode vibrations, whereby the target vibration modes to be suppressed could be selected.

To determine the effectiveness of the hybrid method, numerical simulations and experiments using a 10-bay truss structure were conducted. Regarding energy consumption, the hybrid method showed a significant improvement over the bang–bang active method. Regarding vibration suppression performance, the hybrid method outperformed the energy-recycling semiactive method in suppressing various kinds of vibration. The numerical simulations showed that the hybrid method were more robust to model errors than the bang–bang active method. The experiments under realistic conditions showed that the hybrid method effectively suppressed vibrations excited by a random force as well as single-mode and multiple-mode vibrations.

The sophisticated method to prevent control chattering also helps to reduce the consumption of external energy. This method can be applied to the hybrid method as well as to the energy-recycling semiactive method. It monitors the voltage of piezoelectric transducers instead of imposing a threshold on the parameters. Therefore, it avoids a trial-and-error process to determine the threshold, and it flexibly responds to complicated vibrations, such as random excitations. Its effectiveness was experimentally confirmed.

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